

Electroweak Dyons

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Abstract:

We consider dyon configurations in the standard electroweak model. In the presence of a θ term and no fermions, the usual result $q = (n + \theta/2\pi)e$ is obtained for the electric charge spectrum. The effect of including standard model fermions is discussed qualitatively.

1. Introduction

It is often not appreciated that the standard model of the electroweak interactions contains magnetic monopoles¹. The only way in which an electroweak monopole is different from a usual monopole is that it is always connected by a string to an antimonopole. But this string is *not* a Dirac or other string that surreptitiously returns the magnetic flux; instead it is an electromagnetically neutral string made up of Z magnetic field². And, as originally asserted by Nambu, electroweak monopoles are genuine magnetic monopoles for which the divergence of \vec{B} does not vanish.

At the same time, the electroweak monopole is mysterious in some ways. For example, at first sight, the Dirac quantization condition seems to be violated. This is because the magnetic charge on the electroweak monopole is:

$$m = \frac{4\pi}{e} \sin^2 \theta_W \quad (1.1)$$

where, θ_W is the Weinberg angle and e the electric charge of the electron. In contrast to the Dirac quantization condition, $em \neq 4\pi n$ for any integer n for general θ_W .

In addition, if one generalizes the electroweak monopole to electroweak dyons, a naive analysis leads one to expect fractional electric charge on the dyon even in the absence of CP violation. To see this, one uses the quantization condition relevant for two dyons with magnetic charges m_1, m_2 and electric charges q_1, q_2 :

$$q_1 m_2 - q_2 m_1 = 4\pi n . \quad (1.2)$$

If we consider a monopole and an antimonopole, such as would be present at the ends of a string, together with eqn. (1.1), we get,

$$q_1 + q_2 = \frac{ne}{\sin^2 \theta_W} . \quad (1.3)$$

Disregarding the fermions in the theory, CP is conserved, and dyons with charge $(m_1, -q_1)$ also exist. Then (1.3) together with the $q_1 \rightarrow -q_1$ version of (1.3), gives the electric charge spectrum of the dyon:

$$q = n \frac{e}{\sin^2 \theta_W} , \quad \left(n + \frac{1}{2} \right) \frac{e}{\sin^2 \theta_W} . \quad (1.4)$$

(If we assume that $q = 0$ is in the spectrum, the half integer solutions will be eliminated.)

Hence it appears as though the dyon spectrum is not quantized in units of e .

As first described by Witten³, on including a θ term in the bosonic sector of a theory with magnetic monopoles, the monopoles get an electric charge $e\theta/2\pi$. Since electroweak monopoles are genuine magnetic monopoles, we expect the same phenomenon to occur and electroweak monopoles to get Witten's value of the electric charge. However, if we were to simply add $e\theta/2\pi$ to the charge that appears in (1.4), the periodicity under $\theta \rightarrow \theta + 2\pi$ would be lost. Clearly, something is amiss.

In this paper, we would like to address some of these issues. To start with we outline the electroweak monopole system. In Sec. 2, one of our main goals is to emphasize Nambu's assertion that electroweak monopoles should be thought of as genuine monopoles. We then turn to the construction of dyons at a classical level in Sec. 3. For this we exhibit field configurations that satisfy the asymptotic field equations and have dyonic field strengths. At the classical level, the dyon can have any electric charge but, by quantizing the angular momentum, we expect to get a discrete dyon charge spectrum. This mission is accomplished in Sec. 4 where we find that the electric charge on the dyon is quantized in units of e as one would have hoped for and not as eqn. (1.4) would have us believe. We also include a θ term in the action (without fermions) and find that electroweak dyons carry the Witten value of electric charge exactly as ordinary monopoles do. In Sec. 5 we discuss what might happen to the dyon charge spectrum due to CP violation in the

standard model. Finally we explain the significance that the Witten electric charge on an electroweak monopole may have in the context of electroweak baryogenesis.

2. Electroweak Monopoles

In this section, we will repeat some of Nambu's arguments and derive the magnetic charge on an electroweak monopole. For this, assume that we have a semi-infinite string along the $-z$ axis and a monopole at the origin.

First of all, we define the Z and A gauge fields for general Higgs field Φ :

$$Z_\mu \equiv \cos\theta_W n^a W_\mu^a - \sin\theta_W Y_\mu, \quad A_\mu \equiv \sin\theta_W n^a W_\mu^a + \cos\theta_W Y_\mu, \quad (2.1)$$

where, W_μ^a and Y_μ denote the $SU(2)$ and $U(1)$ hypercharge gauge fields and

$$n^a \equiv -\frac{\Phi^\dagger \tau^a \Phi}{\Phi^\dagger \Phi}. \quad (2.2)$$

There are several different choices for defining the electromagnetic field strength but, following Nambu, we choose:

$$A_{\mu\nu} = \sin\theta_W n^a W_{\mu\nu}^a + \cos\theta_W Y_{\mu\nu} \quad (2.3)$$

where, $W_{\mu\nu}^a$ and $Y_{\mu\nu}$ are field strengths. (The different choices for the definition of the field strength agree in the region where $D_\mu \Phi = 0$ where D_μ is the covariant derivative operator.) And the combination of $SU(2)$ and $U(1)$ field strengths orthogonal to $A_{\mu\nu}$ is defined to be the Z field strength:

$$Z_{\mu\nu} = \cos\theta_W n^a W_{\mu\nu}^a - \sin\theta_W Y_{\mu\nu}. \quad (2.4)$$

The A and Z magnetic fluxes through a spatial surface will be denoted by F_A and F_Z and these are given in the usual way by the surface integrals of the field strengths. Therefore we can write

$$F_Z = \cos\theta_W F_n - \sin\theta_W F_Y, \quad F_A = \sin\theta_W F_n + \cos\theta_W F_Y, \quad (2.5)$$

where we have denoted the $SU(2)$ flux (parallel to n^a in group space) by F_n and the hypercharge flux by F_Y .

Now consider a large sphere Σ centered on the monopole. The monopole-string configuration is such that there is only an electromagnetic magnetic flux through Σ except at the South Pole (S), where there is only a Z magnetic flux. Hence, denoting the electromagnetic flux by F_A and the Z flux by F_Z ,

$$F_Z|_{\Sigma} = 0 = F_A|_S . \quad (2.6)$$

Equation (2.6) together with (2.5) tells us that

$$F_n|_{\Sigma} = \tan\theta_W F_Y|_{\Sigma} , \quad F_n|_S = -\cot\theta_W F_Y|_S . \quad (2.7)$$

But, the hypercharge flux must be conserved as it is a $U(1)$ flux. So

$$F_Y|_{\Sigma} = -F_Y|_S \equiv F_Y , \quad (2.8)$$

and, inserting this and (2.7) in (2.5) gives

$$F_A|_{\Sigma} = \frac{F_Y}{\cos\theta_W} , \quad F_Z|_S = \frac{F_Y}{\sin\theta_W} . \quad (2.9)$$

To proceed further, we need to put in some dynamics. This is most simply done by realizing that the string along the $-z$ axis is an ordinary Nielsen-Olesen vortex⁴ embedded in the electroweak model⁵ and so the flux is quantized in units of $4\pi/\alpha$ where $\alpha \equiv \sqrt{g^2 + g'^2}$ is the coupling of the Z to the Higgs field. Therefore, for the unit winding string,

$$F_Z|_S = \frac{4\pi}{\alpha} . \quad (2.10)$$

Then (2.9) yields,

$$F_Y = \frac{4\pi}{\alpha} \sin\theta_W , \quad F_A|_{\Sigma} = \frac{4\pi}{\alpha} \tan\theta_W = \frac{4\pi}{e} \sin^2\theta_W \quad (2.11)$$

where, $g = \alpha \cos \theta_W = e / \sin \theta_W$.

It is instructive to work out the magnetic flux for the $SU(2)$ fields. From (2.7) with (2.11), the net non-Abelian flux is:

$$F_n = F_n|_S + F_n|_\Sigma = \frac{4\pi}{g} \quad (2.12)$$

just as we would expect for an ordinary $SU(2)$ monopole. That is, the Dirac quantization condition works perfectly well for the $SU(2)$ field and the monopole charge is quantized in units of $4\pi/g$. Another way of looking at (2.12) is to say that the electroweak monopole is a genuine $SU(2)$ monopole in which there is a net emanating $SU(2)$ flux. The structure of the theory, however, only permits a linear combination of this flux and hypercharge flux to be long range and so there is a string attached to the monopole. But this string is not a Dirac string that surreptitiously returns the monopole flux; at best it is a Dirac string only for the hypercharge part of the monopole field. And so the electroweak monopole is a genuine topological monopole in the $SU(2)$ sector of the model. In particular, the magnetic charge on the monopole is conserved and electroweak monopoles can only disappear by annihilating with antimonopoles.

3. Electroweak Dyons

We now show that the electroweak model also admits dyon configurations. To this end, we will write down dyonic configurations that solve the asymptotic field equations. Our analysis is similar to that in Ref. 6, the only difference being that whereas Alford et. al. linearized their equations, we can work with the full non-linear equations. It should also be mentioned that Nambu recognized the possibility of generalizing the monopole solution with what he called “external” potentials. Essentially, the dyon solution is an electroweak monopole together with a particular external potential.

The classical field equations of motion for the bosonic sector of the standard model of the electroweak interactions are:

$$D^\mu D_\mu \Phi + 2\lambda \left(\Phi^\dagger \Phi - \frac{\eta^2}{2} \right) \Phi = 0 \quad (3.1)$$

$$D_\nu W^{\mu\nu a} = j^{\mu a} = \frac{i}{2}g \left[\Phi^\dagger \tau^a D^\mu \Phi - (D^\mu \Phi)^\dagger \tau^a \Phi \right] \quad (3.2)$$

$$\partial_\nu Y^{\mu\nu} = j^{\mu Y} = \frac{i}{2}g' \left[\Phi^\dagger D^\mu \Phi - (D^\mu \Phi)^\dagger \Phi \right] \quad (3.3)$$

where,

$$D_\mu \Phi = \left[\partial_\mu - \frac{i}{2}g W_\mu^a \tau^a - \frac{i}{2}g' Y_\mu \right] \Phi \quad (3.4)$$

and,

$$D_\nu W^{\mu\nu a} = \partial_\nu W^{\mu\nu a} + g\epsilon^{abc} W_\nu^b W^{\mu\nu c} . \quad (3.5)$$

Denoting Nambu's monopole-string configuration by $(\bar{\Phi}, \bar{W}_\mu^a, \bar{Y}_\mu)$, the explicit fields for the monopole in the asymptotic region are:

$$\bar{\Phi} = \frac{\eta}{\sqrt{2}} \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix} \quad (3.6a)$$

where, θ and ϕ are spherical coordinates centered on the monopole. The gauge field configuration in the asymptotic region is:

$$g\bar{W}_\mu^a = -\epsilon^{abc} n^b \partial_\mu n^c + i\cos^2\theta_w n^a (\Phi^\dagger \partial_\mu \Phi - \partial_\mu \Phi^\dagger \Phi) \quad (3.6b)$$

$$g'\bar{Y}_\mu = -i\sin^2\theta_w (\Phi^\dagger \partial_\mu \Phi - \partial_\mu \Phi^\dagger \Phi) \quad (3.6c)$$

where, n^a is given in (2.2) and has the two very useful properties:

$$n^a n^a = 1 , \quad (n^a \tau^a + \mathbf{1})\Phi = 2\mathbf{Q}\Phi = 0 \quad (3.7)$$

where, $\mathbf{1}$ is the 2×2 unit matrix and \mathbf{Q} is the generator for the electromagnetic gauge transformations and is defined to annihilate Φ .

In writing (3.6), there is a subtlety which we should point out. One is used to thinking of a monopole where the non-Abelian nature of the fields is important only in a small core region and the non-Abelian excitations fall off exponentially fast outside this core. Hence, the asymptotic region in this case would be the region outside the core. Here, however, there is a string sticking out of the monopole and there is always a fraction of the region far from the monopole in which Φ does not lie on the vacuum manifold. As we go further away from the monopole, this fractional volume decreases and our asymptotic approximations become valid over a larger fraction of the asymptotic sphere. But the region where our approximations are invalid only diminishes as a power law and not exponentially. However, since we have introduced the denominator in (2.2), n^a has unit magnitude everywhere - even inside the string. In addition, the derivatives of n^a inside the string fall off as $1/R$ where R is the distance from the monopole and n^a can be taken to be constant inside the string in the asymptotic region.

Now we make an ansatz that will describe an electroweak dyon connected by a semi-infinite Z string. This is:

$$\Phi = \bar{\Phi} \quad (3.8)$$

$$W_\mu^a = \bar{W}_\mu^a - \delta_\mu^t \frac{n^a \dot{\gamma}}{\cos\theta_W} \quad (3.9a)$$

$$Y_\mu = \bar{Y}_\mu - \delta_\mu^t \frac{\dot{\gamma}}{\sin\theta_W} \quad (3.9b)$$

where, $\gamma = \gamma(\vec{x}, t)$ and overdots denote partial time derivatives.

We can work out the field strengths for the dyon-string ansatz. The change in the field strengths are:

$$\delta W_{ij}^a = 0 = \delta Y_{ij} \quad (3.10)$$

$$\delta W_{it}^a = -\frac{\bar{D}_i(n^a \dot{\gamma})}{\cos\theta_W}, \quad \delta Y_{it} = -\frac{\partial_i \dot{\gamma}}{\sin\theta_W} \quad (3.11)$$

where,

$$\bar{D}_\mu(n^a\dot{\gamma}) \equiv \partial_\mu(n^a\dot{\gamma}) + g\epsilon^{abc}\bar{W}_\mu^b(n^c\dot{\gamma}) \quad (3.12)$$

is the covariant derivative of $n^a\dot{\gamma}$ with the monopole-string gauge field of eqn. (3.6).

Now we first check if the Φ eqn. (3.1) is satisfied by the dyon-string ansatz in the asymptotic region. For this, we use (3.7) and find that

$$D_\mu\Phi = \bar{D}_\mu\bar{\Phi} \quad (3.13)$$

where, \bar{D}_μ is defined as in (3.4) but with barred gauge fields. This leads to

$$D^\mu D_\mu\Phi = \bar{D}^\mu \bar{D}_\mu\bar{\Phi} \quad (3.14)$$

upon using $\bar{D}_t\bar{\Phi} = 0$. Then it trivially follows that the Φ equation of motion with the dyon-string ansatz is satisfied, if it is satisfied by the monopole-string ansatz.

Next, we work out the change in the currents due to the extra terms in (3.9). A little algebra yields

$$\delta j^{\mu a} = 0 = \delta j^{\mu Y} . \quad (3.15)$$

Inserting these in (3.2) and (3.3) we find that the W gauge field equation leads to:

$$\bar{D}^i \bar{D}_i(n^a\dot{\gamma}) = 0 \quad (3.16)$$

$$\partial_t \bar{D}^i(n^a\dot{\gamma}) = 0 \quad (3.17)$$

while the Y equation gives:

$$\partial^i \partial_i \dot{\gamma} = 0 \quad (3.18)$$

$$\partial_t \partial^i \dot{\gamma} = 0 . \quad (3.19)$$

The first two equations ((3.16) and (3.17)) of these four equations for γ , are consistent with the second two when we use the explicit expressions in (3.6) for the monopole-string

configuration. To see this, we use (3.6) and obtain

$$\bar{D}_i(n^a \dot{\gamma}) = n^a \partial_i \dot{\gamma} . \quad (3.20)$$

With this relation, (3.16) and (3.17) reduce to (3.18) and (3.19).

In obtaining (3.20) we have made use of the explicit form of the monopole-string field configuration (eqn. (3.6)) for the first time. and the question arises if (3.20) holds within the string too where (3.6) does not apply. This is indeed the case because n^a is constant and \bar{W}_μ^a is parallel to n^a inside the string in the asymptotic region (see the discussion following (3.7)). Using these facts, n^a pulls out of the partial derivative in \bar{D}_i and the cross-product term in the covariant derivative (eqn. (3.12)) does not contribute. So (3.20) is valid even inside the string.

Next, we use separation of variables and write

$$\gamma = \xi(t)f(\vec{x}) . \quad (3.21)$$

This leads to

$$\ddot{\xi} = 0 , \quad \nabla^2 f = 0 , \quad (3.22)$$

with general solutions that can be found in many text-books⁷. The particular solution that we will be interested in is the solution that gives a dyon. Hence, we take:

$$\xi = \xi_0 t , \quad f(r) = -\frac{q \sin \theta_W \cos \theta_W}{4\pi \xi_0} \frac{1}{r} , \quad (3.23)$$

where, ξ_0 is some constant. Now, using (2.3), together with (3.7), (3.9) and (3.23) we then get the dyon electric field:

$$\vec{E}_A = \frac{q}{4\pi} \frac{\vec{r}}{r^3} . \quad (3.24)$$

For a long segment of string, the monopole and the antimonopole at the ends are well separated and we can repeat the above analysis for both of them independently. Therefore,

the electric charge on the antimonopole at one end of a Z string segment is uncorrelated with the charge on the monopole at the other end of the string. This means that we can have dyons of arbitrary electric charge at either end of the string.

This completes our construction of the dyon-string system in the electroweak model. As of now, the charge q on the dyon is arbitrary; we will quantize it in the next section.

4. Charge Quantization

Consider a segment of Z string which has electroweak dyons at either end with magnetic and electric charges

$$(m, q_1) \quad \text{and} \quad (-m, q_2) . \quad (4.1)$$

The angular momentum of this system is given by the sum of the angular momenta in the $SU(2)$ and hypercharge fields. However, according to our discussion in Sec. 2, the monopole resides entirely in the $SU(2)$ sector and so there can be no angular momentum due to the hypercharge field. Furthermore, the dyons have $SU(2)$ magnetic and $SU(2)$ electric charges given by

$$\left(\frac{4\pi}{g}, \frac{q_1}{\sin\theta_W} \right) \quad \text{and} \quad \left(\frac{-4\pi}{g}, \frac{q_2}{\sin\theta_W} \right) . \quad (4.2)$$

And the $SU(2)$ fields due to these charges are long range since the fields occur as part of the electromagnetic flux from the monopole. So the angular momentum in the field is given by the usual “cross-product” of the magnetic and electric charges:

$$L = \frac{4\pi}{g} \frac{q_2}{\sin\theta_W} - \frac{-4\pi}{g} \frac{q_1}{\sin\theta_W} = \frac{4\pi(q_1 + q_2)}{g\sin\theta_W} \quad (4.3)$$

Quantizing the angular momentum in units of 4π ($\hbar = 1$) and using $e = g \sin\theta_W$ yields

$$q_1 + q_2 = ne \quad (4.4)$$

where, n is an integer.

The charges q_1 and q_2 are independent of each other and if we have CP invariance, we can consider a CP transformed dyon on one end of the string. This will change q_2 to $-q_2$ and hence, the charge on each of the dyons is quantized as ne or $(n + 1/2)e$ if our theory is invariant under CP. If we further assume that the zero electric charge dyon is in the spectrum, then the quantization is in units of e .

One way to have CP violation in the theory is to have a θ term in the action. For the standard model of the electroweak interactions (without fermions!) this would be

$$S_\theta = \frac{g^2\theta}{32\pi^2} \int d^4x W_{\mu\nu}^a \tilde{W}_{\lambda\sigma}^a = \frac{g^2\theta}{8\pi^2} \int d^4x \vec{E}^a \cdot \vec{B}^a . \quad (4.5)$$

Now, in the asymptotic region, only the $SU(2)$ fields have a net flux from the monopole (see (2.12)). So we can write:

$$\nabla \cdot \vec{B}_n = \frac{4\pi}{g} \delta^3(\vec{x}) . \quad (4.6)$$

where, the subscript n denotes a combination of $SU(2)$ field strengths parallel to n^a . Following Coleman's derivation⁸ of the Witten effect, an integration by parts of (4.5) then shows that the dyon acquires an extra $SU(2)$ charge which is $g\theta/2\pi$. But this is exactly an electric charge $e\theta/2\pi$ where $e = g \sin\theta_W$ (see (2.1)). Hence, the electric charge on the dyon in the presence of a θ term is

$$q = \left(n + \frac{\theta}{2\pi} \right) e . \quad (4.7)$$

This agrees fully with the standard result for dyons and it is reassuring to see that the periodicity under $\theta \rightarrow \theta + 2\pi$ is intact.

It should be mentioned that, even though the electric charge on an electroweak dyon can be fractional as in (4.7), the total electric charge on the dyon-string system is always integral because the CP violating fractional charge on the monopole is equal and opposite to that on the antimonopole.

5. Discussion

The standard model contains fermions in addition to the bosonic sector that we have considered so far. With the inclusion of fermions it is known that the θ term can be gauged away and so the Witten charge must disappear. However, it is also known that the standard model contains CP violation in the fermionic sector via the Kobayashi-Maskawa matrix. As suggested by Witten, any CP violation in a model with monopoles will feed into the dyon charge spectrum. Hence, electroweak monopoles should feel the KM matrix and obtain a fractional charge. While the existence of such a fractional charge seems believable, it is a much harder problem to actually compute what the value of the charge is. This burning question is currently being investigated.

The presence of charge on electroweak monopoles is likely to have a CP violating influence on the evolution of electroweak strings. In connection with our earlier finding that electroweak strings can carry baryon number⁹, the CP violating dynamics of such strings could lead to the potentially important phenomenon that the decay of baryon number carrying strings preferentially produces baryons rather than antibaryons.

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